through the permeable wall the temperature of the latter decreases with increase in Re, and at sufficiently high Re the temperature of the wall and the liquid traveling toward it coincide.

The results of the numerical experiments indicate the effectiveness of the difference method described over a wide range of Reynolds and Grashof numbers.

NOTATION

x, y, Spatial coordinates; τ, time; u, v, projections of velocity vector on x and y axes; ψ, flow function; ω, vorticity; t, temperature; g_x, g_y, projections of acceleration created by external mass forces along x and y axes; β, temperature expansion coefficient; ν, λ, α, a, coefficients of kinematic viscosity, thermal conductivity, heat liberation, and thermal diffusivity; ρ, c, density and specific heat of liquid; t_m, temperature of external medium surrounding channel wall.

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A NUMERICAL METHOD OF CALCULATING THE BOUNDARY OF STABILITY OF THERMALLY INDUCED ACOUSTIC OSCILLATIONS

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We present the results of a theoretical and experimental study of the conditions under which thermally induced acoustic oscillations arise in nonisothermal pipelines of variable cross section.

In cryogenic nonisothermal pipelines, closed at the warm end and open at the cold end, thermally induced oscillations can arise, accompanied by a large heat flux in the low temperature zone. The stability boundary of such oscillations determines the conditions under which they arise and it depends on the wall temperature profiles of the pipeline and its cross section. For the case in which the temperature distribution and the pipeline cross section along its length are specified by single-step functions, a stability analysis was given in [1-4]. In [5] a numerical study was made of the influence of the temperature profile on the stability of the oscillations. In the persent paper we solve the very same problem, but for a pipeline of variable cross section.

The set of equations with respect to the amplitude for the oscillations of a gas with a frequency ω in a nonisothermal tube of variable cross section has the form [4].

$$i\omega\rho + \frac{\rho_0}{r_0^2(x)}\frac{\partial}{\partial x}(r_0^2(x)U) + \frac{\partial\rho_0}{\partial x}U = 0, \quad (1)$$

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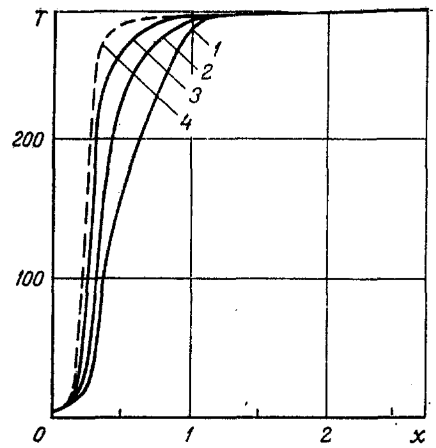

Fig. 1

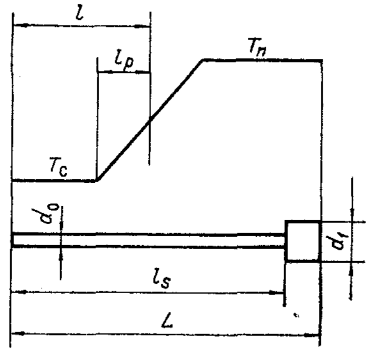

Fig. 2

Fig. 1. Experimental temperature profiles. Oscillational parameter values are shown in Table 1.

Fig. 2. Temperature and cross-sectional profiles.

TABLE 1. Calculated and Experimental Results

Temp. profile number	Expt.	1	2	3	4
Pipeline diam., mm	Calc.	2,7	3,2	3,7	4,0
	Expt.	3,4	3,4	3,4	3,4
Freq., Hz	Calc.	27,8	28,1	28,0	28,2
	Expt.	22,7	26,3	26,3	No oscillation
Ampl. of pressure oscillation, N/m²	Expt.	2400	1500	900	No oscillations

$$i\omega U+\frac{1}{\rho_0}\frac{dP}{dx}=\nu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right), \tag{2}$$

$$i\omega\left(T-\frac{P}{\rho_0 c_p}\right)+U\frac{dT_0}{dx}=\frac{\lambda}{\rho_0 c_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right). \tag{3}$$

The system is completed by the addition of the linearized equation of state

$$T=\frac{P}{R\rho_0}-\frac{T_0}{\rho_0}\rho. \tag{4}$$

This system may be reduced to a single equation for the amplitude of the pressure P, whose solution over a section of the pipeline with a constant temperature and a constant cross section is the following:

$$P=A\sin kx+B\cos kx. \tag{5}$$

In [3] the following junction conditions were determined at a point of discontinuity $x = l$ for the temperature and pipeline cross section functions:

$$P(l-0)=P(l+0); \tag{6}$$

$$\psi(l-0)=\psi(l+0), \tag{7}$$

where

$$\psi=\frac{G}{k^2}\frac{dP}{dx};\quad G=r_0^2(x)\,g_1 E,$$

and k_1, g_1 and E are functions of the complex variable $\eta=(i\omega/\nu)^{1/2}r_0(x)$.

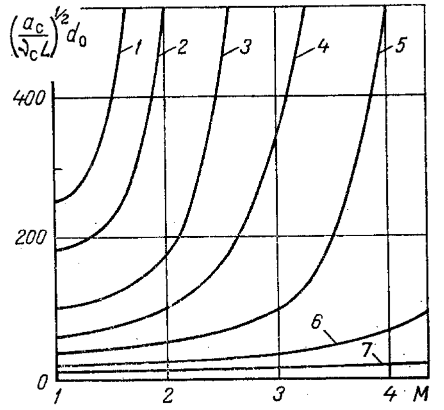

Fig. 3

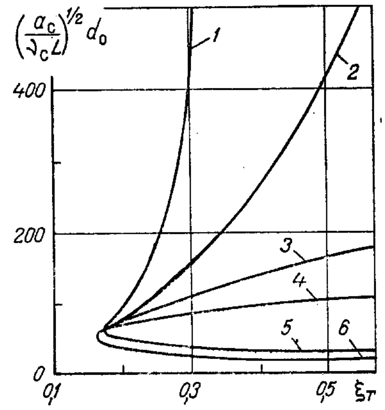

Fig. 4

Fig. 3. Influence of the cross-sectional ratio M on the stability boundary for various slopes S_p of the nonisothermal section. $\xi_T = 0.5$; $\xi_S = 0.95$; $T_n/T_c = 50$. Curves 1-6 correspond to the upper branch of the stability boundary, curve 7 corresponds to the lower branch: 1) $S_p = 0.4$; 2) 0.6; 3) 0.8; 4) 0.9; 5) 0.95; 6) 1.0; 7) 0.8.

Fig. 4. Influence of the location of the nonisothermal section on the stability boundary for various cross section ratios: $T_n/T_c = 50$; $S_p = 0.8$; $\xi_S = 0.95$. Curves 1-4 are for the upper branch, curves 5-6 for the lower branch: 1) M = 3.0; 2) 2.5; 3) 2.0; 4) 1.0; 5) 1.0; 6) 3.0.

The numerical method we present for solving the oscillational stability problem is a further extension of the method used in [4] and consists in the following. The pipeline length is divided into N intervals on which the temperature and cross-sectional profiles are approximated by discrete piecewise-constant functions with discontinuities at the ends. The pressure amplitude distribution inside these intervals is determined by sectional functions of the form (5); the conditions (6) and (7) are satisfied at their endpoints. Upon satisfying these conditions, we obtain a system of 2N equations with respect to their coefficients. The stability boundary is obtained by setting the determinant of this system equal to zero.

To verify our method we checked our results experimentally and we also compared them with the analytical results given in [1-4] and the numerical solutions given in [5].

As experimental element we chose a tube with internal diameter d_0 = 3.4 mm and length L = 2650 mm, which had at its end an axisymmetric capacity defined by its diameter d_1 = 14 mm and its length H = 100 mm. The tube was placed in a cryostat with liquid helium. Different wall temperature profiles were created due to the variation in the volumetric flow of the gas directed along the tube. The tube temperature was measured by thermocouples and impedance thermometers. The tube wall temperature profiles, obtained in the experiment, are shown in Fig. 1. The amplitude of the pressure oscillations corresponding to these profiles decreases as the ordinal number increases. Profiles with steeper transitions from low to high temperature, as can be seen from the figure, stand close to the boundary of the domain of oscillations. At first glance this contradicts the general understanding, according to which an increase in the temperature gradient must lead to an amplification of the oscillations. However, in the given case, simultaneously with an increase in the gradient there is a reduction in the low temperature section, and the latter is the reason for the vanishing of the oscillations. Pressure oscillations were recorded by means of a DD6S pressure transmitter. A comparison of the calculated and experimental data was made in the following way. An experimentally determined temperature profile was given and then two boundary values for the diameter were calculated, which define an interval in which oscillations exist for a given profile. When oscillations are present the pipeline diameter must fall in this interval, and it must fail to do so in the absence of oscillations. The experimental and calculated values are shown in Table 1, wherein only the minimum values of the pipeline diameter are given for which oscillations are possible. The maximum values extend over several tens of millimeters and more and are not supplied here. According to our calculations the oscillations must die away for a pipeline diameter lying between 3.7 mm and 4.0 mm. For a pipeline diameter of 3.4 mm the deviation amounts to 10-20%, and, in our view, this lies within the allowable boundaries.

As an example, we consider the problem concerning the influence of the capacity at the warm end of a pipeline on the stability of the oscillations. It was shown in [4] that the influence of the capacity has a dual nature. The capacity can contribute to an excitation of the oscillations and it also can contribute to their attenuation. At the same time, in cryogenic technology, nonisothermal pipelines frequently involve elements of small volumetric capacity (pressure transmitters, valves, and vents). We studied the stability boundary of oscillations for a pipeline with a small axisymmetric capacity, having a piecewise-linear temperature profile (Fig. 2). The slope of the temperature profile is characterized by the parameter $S_p = (l - l_p)/l$, and its position by $\xi_T = l/L$. The stability boundary (Figs. 3 and 4) has two branches, which bound a domain of instability from above and below. Moreover, the position of the lower branch depends weakly on the slope of the temperature profile and the magnitude of the expansion at the end of the pipeline. The influence of these factors on the upper branch is substantial. As M increases (Fig. 3) the upper branch of the stability boundary turns upwards, and the more steeply it does so the larger the slope of the nonisothermal section of the temperature profile. Consequently, under these conditions, the capacity expands beyond the domain of instability, i.e., it contributes to excitation of the oscillations. The effect of capacity on the stability boundary also depends on the location of the nonisothermal section, which is defined by the parameter ξ_T (see Fig. 4). As the nonisothermal section is displaced towards the cold end, the upper branch of the boundary is lowered while the lower branch is raised, thus narrowing the domain of instability. Moreover, the lower boundary for a pipeline without capacity (M = 1) lies below that for a pipeline with capacity (M = 3). This means that in this case the capacity attentuates the oscillations.

NOTATION

x, r, axial and radial coordinates; ρ_0, P_0, T_0, mean values of density, pressure and temperature; ρ, P, T, U, amplitudes of density, pressure, temperature and velocity fluctuations; λ, gas thermal conductivity; c_p, isobaric heat capacity; σ, Prandtl number; ν, kinematic viscosity of gas; a, sound velocity; $r_0(x)$, $T_0(x)$, radius and temperature of the pipeline in the cross section x; T_c, wall temperature at the cold end; d_0, pipeline diameter; d_1, diameter of the tank; $M = d_1/d_0$; ω, fluctuation frequency; k_1, g_1, E, functions of temperature and cross sections determined in [4]. Index c, values of parameters at temperature T_c.

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